Making Sense of Word Problems:

Strategies for Problem Solving in Secondary Mathematics

Introduction

Mathematics is the art of problem solving, yet it is traditionally taught in a way that emphasizes computation. By the time they get to high school, students expect to memorize a few formulas and plug in numbers to find an answer. They are quickly frustrated when they find that following a set of steps is not always the way to a solution. Part of the problem is that textbooks emphasize procedure, but don't teach students how to translate word problems (Edens & Potter, 2008). Instructors must find ways to make word problems less intimidating to students. Some strategies include helping students understand the vocabulary and structure of word problems, visually representing information, and using number lines.

Understanding the Structure

Mathematics has long been considered a universal language. If math is considered a language this would then imply that to be successful in mathematics students need to understand the mathematics vocabulary just as they would for any other language. Leali, Byrd, and Tungmala (2012) state that mathematics content vocabulary has a lot of overlap. For example, the word "altogether" might be used to indicate either addition or multiplication. Furthermore, operations such as addition can be represented by words like "plus", "increased by", or "sum". Word problems can be confusing for students because it is not always clear what mathematical operation should be performed. Students must use a higher level of thinking to figure that out for themselves. Tim Anglen, who has been teaching mathematics at the secondary level for 17 years, agrees that it is imperative to teach these keywords. He utilizes a poster in his classroom to remind students of these keywords when they are solving word problems (T. Anglen, personal communication, December 22, 2017).

Mathematical Terms

Knowing mathematical terms is important, but students can often be confused by words beyond their everyday vocabularies. Changing the text does not change the concept of the problem. "But rather, it reduces the readability demands by eliminating linguistic characteristics that get in the way of comprehensions" (Kurtz, Gómez, & Jiminez-Silva, 2017, p. 34). When a student is still working on a concept, such as solving an equation, it is not necessary to make that task more difficult by including difficult words in the word problem. Students could overthink what they are being asked to do if they do not understand some of the vocabulary. To help with the readability of a word problem it is suggest to read the problem aloud together. This could be done as a class or during one-on-one work time (Kurtz et al., 2017). The read aloud might be followed by a discussion about what certain words mean. To further simplify the problem students can replace numeric words with numbers and break up the problem into a list of facts, rather than keeping it in paragraph form (Kurtz et al., 2017). These seemingly simple tasks can make a world of difference for students who struggle with word problems.

Word Problems

Also imperative to breaking down a word problem is understanding the basic structure of a word problem. All word problems have a premise and conclusion, which is often the question, or what the students is being asked to find out (Vacaretu, 2008). In a study that took place in a 9th grade Romanian classroom the instructor asked students to identify different parts of a word problem. The "objective for this lesson was to help students understand what constitutes insufficient, sufficient, and redundant information in a mathematics problem… [the teacher] noticed that students, when solving problems based on real-life situations, often got sidetracked by irrelevant details while ignoring relevant information" (Vacaretu, 2008, p. 454). In a series of activities students in this math class practiced identifying what they were being asked to find. They looked for pieces of information they needed and decided what could be dismissed (Vacretu, 2008). To further simplify the problem students can replace numeric words with numbers and break up the problem into a list of facts, rather than keeping it in paragraph form (Kurtz et al., 2017). These seemingly simple tasks can make a world of difference for students who struggle with word problems.

Taking the time to break down a word problem is necessary for a student to solve the problem accurately. It prevents what Edens and Potter (2008) call “compute first and think later” or “number grabbing mistakes” students often make (p. 185). Instead of taking the time to think through what they are being asked to do, students might take the numbers they are given and plug them into a formula that is familiar to them. Edens and Potter (2008) found the following:

> Unsuccessful problem solvers attempt what they term a ‘short-cut approach’ by translating the key propositions in the problem statement to a set of computations that will produce an incorrect answer. For example, a key word such as ‘more than’ is automatically interpreted as requiring addition. Thus, the problem solver does not accurately represent the situation described in the problem. (p. 185)

In the situation described above “more than” could mean addition, but it could also indicate the problem is about an inequality. For this reason it is important for students to have a strong understanding of mathematical key terms and information that is relevant to the problem.

Visual Representations

As early as primary grades students have been instructed to “draw a picture” as the first step to solving a problem. Research has shown that this strategy is only useful when done correctly. Many students do not know how to draw a picture that will be beneficial. Perhaps the first step is to change the language that is used. The word "diagram" is the better word choice. "Diagrams have often been cited as a powerful visualization strategy for representing a word problem, as they can be used to help unpack the structure of a problem and thus lay the foundation for its solution, simplify a complex situation, and make abstract concepts more concrete and, as a result, familiar" (van Garderen, 2007, p. 540). For a diagram to be a powerful tool students must first learn how to create one.

A useful diagram is one that is primarily schematic versus pictorial. "A schematic drawing may include details, but the details depict a component of the problem, such as key numerals" (Edens & Potter, 2008, p. 186). Pictorial diagrams would likely have far more but unnecessary details. They would likely not include numeric labels. Also, pictorial diagrams may not contain appropriate spatial relations and proportions between the objects drawn (Edens & Potter, 2008). The proportions aspect is especially important when working with a concept such as fractions. When drawing a diagram of a fraction students need to make sure all parts of the fraction are roughly equal. This way the picture is useful to aid in the understanding of the concept as well as helping the student make sense of the problem (T. Anglen, personal communication, December 22, 2017).

In their study Edens and Potter (2008) found that schematic representations were positively related with total problem solving, and there was not correlation between pictorial representations and problem solving . Students' performance did not improve much in van Garderen's study, but overall the students did say they were very satisfied with the strategy and would use it again (van Garderen, 2007).

Van Garderen's study also mentioned the use of "backward chaining". Van Garderen (2007) describes this as:

> an approach that requires the student to first identify the overall primary goal of the problem, or the 'final answer.' With that information, the student is then taught that he or she needs to identify the secondary problem, or 'partial answer' necessary to get the final answer. (p. 545-546).

This appears to be another way of describing what other researchers have done as well. Students must break down the structure of the word problem before they can put the information into a diagram.

Perhaps most importantly, "when students generate their own diagrams, it adds greater meaning to a task, thus leading to a greater likelihood of diagrams' being used in other problem solving situations" (van Garderen, 2007, p. 541). Diagramming is a strategy that can be used over and over, as opposed to a series of steps that only apply in certain situations.

Number Lines

A specific type of schematic diagram, one that has no pictures, is a number line. These are not just used for counting, but are a tool to represent information when problem solving. Their simple, easy to read format allows "for observation of students' thinking processes, and thus teachers can quickly recognize and correct any errors in thinking that may occur" (Gonsalves & Krawec, 2014, p. 162).

As stated previously students must still pick out the important information and determine what is being asked. When using a number line the goal, or question asked, must be represented in the diagram. This can be done using a question mark. If students are looking for the total it may be placed at the top of the line. If there is a missing piece the question mark goes somewhere on the line in relation to the other numbers (Gonsalves & Krawec, 2014). This form of schematic diagram is so useful because students see that each piece of the diagram represents something required to solving their problem. They also need to understand that space is not arbitrary as it represents length (Gonsalves & Krawec, 2014).

It should be noted that schematic diagrams such as number lines are not useful if students have a week understanding of the concepts contained in the word problems. In a study over the use of schematic diagrams by students with varying ability levels van Garderen, Scheuermann, and Jackson found that students with les conceptual understanding were less successful in producing accurate schematic representations (van Garderen, Scheuermann, & Jackson, 2013).

Conclusion

Despite being the most applicable to real life word problems are often pushed to the side. They are tacked on at the end of a lesson when students have lost focus More time is spent teaching the basic concept, so understanding of a word problem is seen as secondary because it is a more advanced skill. Using strategies such as reviewing vocabulary and categorizing information, drawing diagrams, and creating number lines can make word problems less intimidating for students. These tools can equip them to be more successful in the math classroom.

References

Edens, K., & Potter, E. (2008). How students “unpack” the structure of a word problem: Graphic representations and problem solving. *School Science & Mathematics*, *108*(5), 184-196.

Gonsalves, N., & Krawec, J. (2014). Using number lines to solve math word problems: A strategy for students with learning disabilities. *Learning Disabilities Research & Practice (Wiley-Blackwell)*, *29*(4), 160-170.

Kurz, T. t., Gómez, C. c., & Jimenez-Silva, M. D. (2017). Guiding preservice teachers to adapt mathematics word problems through interactions with ELLs. *Journal Of Urban Mathematics Education*, *10*(1), 32-51.

Leali, S., Byrd, D. R., & Tungmala, M. (2012). instructional strategies and word problems of english language learners. *Journal Of The International Society For Teacher Education*, *16*(2), 98-109.

Vacaretu, A. P. (2008). Reading texts & writing problems to improve problem solving. *Mathematics Teacher*, *101*(6), 451-455

van Garderen, D. (2007). Teaching students with LD to use diagrams to solve mathematical word problems. *Journal Of Learning Disabilities*, *40*(6), 540-553.

van Garderen, D., Scheuermann, A., & Jackson, C. (2013). Examining how students with diverse abilities use diagrams to solve mathematics word problems. *Learning Disability Quarterly*, *36*(3), 145-160.